LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION –**PHYSICS**

THIRD SEMESTER – **APRIL 2019**

16/17UPH3MC01– MATHEMATICAL PHYSICS

Date: 24-04-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART - A (10 × 2 = 20 MARKS)

ANSWER **ALL** QUESTIONS.

1. Find the complex conjugate of $\frac{2+3i}{1-i}$
2. Define logarithm of a complex number.
3. Find the constants a, b, c so that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational.
4. If a force $\vec{F} = 2x^2y\hat{\imath} + 3xy\hat{\jmath}$ displaces a particle in the *x*-y plane from (0, 0) to (1, 4) along a curve $y = 4x^2$. Find the work done.
5. How does a Fourier series simplify in the case of an odd or even function?
6. State the linearity of Fourier transforms.
7. Express Laplacian in polar and cylindrical coordinates.
8. Write down the D'Alemberts solution of wave equation.
9. Write Simpson's $(1/3)^{rd}$ rule?
10. Write Newton's forward interpolation formula.

PART - B (4 × 7.5 = 30 MARKS)

ANSWER ANY **FOUR** QUESTIONS.

11. If $\cosh x = \sec\theta$, prove that: $\theta = \frac{\pi}{2} - 2tan^{-1}(e^{-x})$
12. Evaluate the following complex integration using Cauchy's integral formula

$$\int_c \frac{3z^2 + z + 1}{(z^2 - 1)(z + 3)} dz$$

where C is the circle $|z| = 2$

13. State and apply Green's theorem to evaluate $\oint_c [(2x^2 - y^2)dx + (x^2 + y^2)dy]$, where C is the boundary of the area enclosed by the *x*-axis and the upper half of circle $x^2 + y^2 = a^2$.

14. Find the complex form of the Fourier integral representation of

$$f(x) = \begin{cases} e^{-kx}, & x > 0 \text{ and } k > 0 \\ 0 & otherwise \end{cases}$$

15. Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming the initial temperature is

$$f(x) = \begin{cases} x & if \quad 0 < x < L/2 \\ L - x & if \quad \frac{L}{2} < x < L \end{cases}$$

16. (a)Using modified Euler's method to compute y(0.1) with h = 0.1 from $y' = y - \frac{2x}{y}, y(0) = 1$

(b) Using Trapezoidal rule evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals.

PART - C (4 × 12.5 = 50 MARKS)

ANSWER ANY **FOUR** QUESTIONS:

17. (a)Find the values of C_1 and C_2 such that the function

$f(z) = x^2 + C_1 y^2 - 2xy + i(C_2 x^2 - y^2 + 2xy)$is analytic. Also find $f'(z)$.

(b)Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2+y^2}$ are harmonic functions of (x, y) but are not harmonic conjugates.

18. Find the directional derivative of $\nabla(\nabla f)$ at the point (1, -2, 1) in the direction of the normal to the surface$xy^2z - 3x + z^2$, where $f = 2x^3y^2z^4$.

19. Find the two half-range expansions of the function

$$f(x) = \begin{cases} \frac{2k}{L}x & if \quad 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L - x) & if \quad \frac{L}{2} < x < L \end{cases}$$

20. Derive the one-dimensional wave equation of a vibrating string.

21. Obtain the Newton's forward interpolating polynomial $P_5(x)$ for the following tabular data and interpolate the value of the function at $x = 0.0045$

x	0	0.001	0.002	0.003	0.004	0.005
y	1.121	1.123	1.1255	1.127	1.128	1.1285

22. Find the smallest positive root of the equation $xe^{-2x} = \frac{1}{2}\sin x$ and corrected to three decimal places using Newton-Raphson method.

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